Exercise 44

Find the inverse function of $f(x) = \frac{1}{x-1}$. Use a graphing utility to find its domain and range. Write the domain and range in interval notation.

Solution

In order to find the inverse function, switch x with y in the given equation.

$$x = \frac{1}{y - 1}$$

Now solve for y.

$$x \times (y-1) = \frac{1}{y-1} \times (y-1)$$
$$xy - x = 1$$
$$xy = 1 + x$$
$$y = \frac{1+x}{x}$$

Therefore, the inverse function is

$$f^{-1}(x) = \frac{1+x}{x}$$

Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 1) \cup (1, \infty)$